Stirling numbers of the second kind triangular set of equations packages array

Based on Algorithm family tree of the Human or Jinni

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Abstract:

This article is a continuation for previous article in title {another look to Stirling numbers ...} in previous article we described about algorithm family tree of the human and jinni for generating stirling numbers of the second kinds specially we described about Human's section but we never describe about jinni section sufficiently

for example there is no any information about root (jinni's prim father) of jinni's sons in previous article

Now by presenting two separated triangular arrays of packages for Human and Jinni in title { Stirling numbers of the 2th kind triangular set of equation packages array based on family tree of the Human and jinni algorithm } we are going to describe about condition of these two creatures for having role in algorithm family tree of the Human and Jinni

As it said before this is only a try for possibility of formulation the Stirling numbers by corresponding an imaginary subject (*Human and Jinni*) with a real object (*Stirling triangular array*) but this kinds of corresponding **by itself** is interest and reviewable

Specially the condition of Jinni's packages are very interest

The author know that the mentioned algorithm is completely imaginary and this is not real manner for generating the children of Humans or Jinni {and this kind of generation perhaps can be find among some strange animals or plants or microorganism in to the earth or other planets} but the important subject is possibility of corresponding two different ruled system as imaginary and real with together

Algorithm family tree in spite of being imaginary is interest and strange and considerable

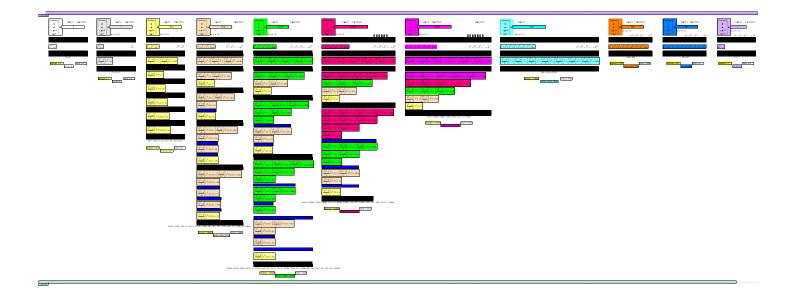
Because the possibility of correspondence an imaginary issue with a math subject as stirling triangular array and also in other stage possibility of determining the terms in to the nth difference sequences in **Stirling magic cube** (*presented in end of fallowing article*)

As a result at least by using of the algorithm family tree easily we can create Stirling numbers sets as below table list for Human packages and in fallow table list for Jinni packages

Keywords:

Stirling numbers; Human and Jinni packages array; Stirling magic cube; algorithm family tree of the Human and Jinni; Stirling 3D array; Stirling Jinni packages; Stirling Human packages





Human's Stirling numbers packages

Human section:

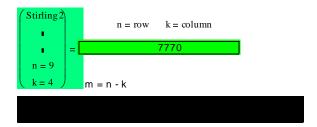
As it said before (another look to Stirling numbers of the second kind) that, each one the Stirling numbers of second kind can be written as one package of two separated kinds of numerical equations (math relations) by the name human's family tree and jinni's family tree now on basis of it we are going to show the Stirling numbers triangular array as two separated table lists

In each one of the mentioned table lists the stirling numbers which locate in a same row make one Stirling set of human's family tree or jinni's family tree that sum of the total values of same human's and jinni's packages is equal with a Stirling number

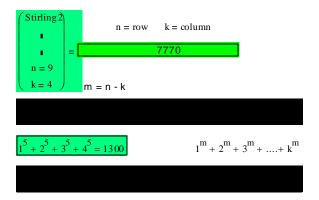
Human packages:

In each one of the Humans packages fist there is a section for identity of related Stirling number

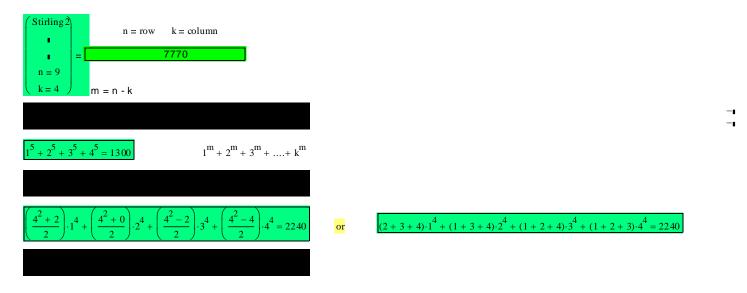
For example identity of Stirling number $\{7770\}$ in Human section is as down it show that the Stirling number $\{7770\}$ is a Second kind of Stirling number locate in row $\{n = 9\}$ and column $\{k = 4\}$ of Human's section



The next section of Human package is sum of the $\{m.^{th}\}$ powers of first $\{n\}$ natural numbers (a pure form of a math relation) that $\{m = n - k\}$ & $\{n = \text{row number}\}$ & $\{k = \text{column number}\}$ for above example $\{7770\}$ is as down figure



The next section of Human's package is human's prim father which for above example {7770} is as down figure



Generation Grade (G.G):

The exponents of the first coefficients of parenthesis are **generation grade** for example the generation grade for above numerical equation (math relation) is 43 and also for down numerical equation are 22 & 23

$$\left(\frac{3^2+1}{2}\right) \cdot 1^2 \cdot 3 \cdot 3 \cdot 3 + \left(\frac{3^2-1}{2}\right) \cdot 2^2 \cdot 3 \cdot 3 \cdot 3 = 567$$

$$\left(\frac{4^2+2}{2}\right) \cdot 1^2 \cdot 4 \cdot 5 + \left(\frac{4^2+0}{2}\right) \cdot 2^2 \cdot 4 \cdot 5 + \left(\frac{4^2-2}{2}\right) \cdot 3^2 \cdot 4 \cdot 5 = 2080$$

Main humans section rule: Reducing of Generation Grade {R.G.G}

In each time of transferring from one generation to next other, one unit of the generation grade will be reduce {R.G.G}

Prim Human's father has two opposite characters

- 1- Because changing the generation to next other first of all he should do {R.G.G}
- 2- **Good and benevolent father**: as a gift he puts the greatest coefficients of his equation as the last coefficients for each one of the terms (he increase the numbers of coefficients for each one of the terms)
- 3- **Bad father for future generation**: as jealousy he eliminate the greatest term of obtained equation from right end of equation

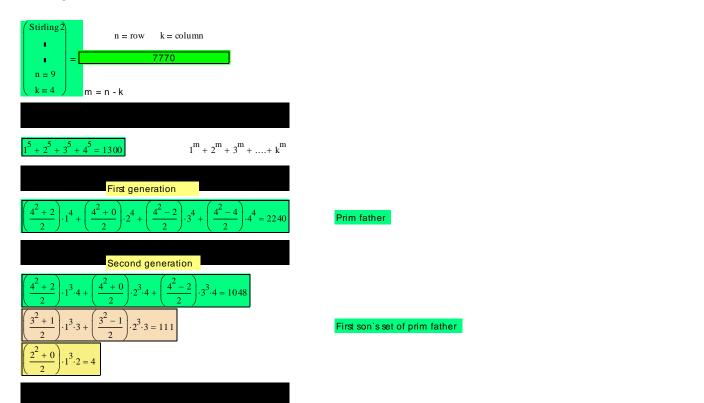
The obtained equation will be the first son of prim father in second generation as below example for {7770}



The prim father by using of the first son's sample and on the basis of below rules generate **the set of his sons** in second generation

- 1- Because changing the generation to next other first he should do {**R.G.G**}
- 2- Eliminate one unit of the last terms of equation step by step of generating the sons
- 3- reduce one unit of each base numbers locate in numerator of parenthesis in terms step by step of generating the sons
- 4- reduces one unit of value of the last coefficients of terms (the first coefficients are invariable and fixed figures) step by step of generating the sons

For the fallowing example Human's package $\{7770\}$ located in $\{n = 9\}$ and $\{k = 4\}$ the set of the sons (*brothers*) are as down figure



After completion the generating all sons in second generation, each one of the generated sons will change to a good and benevolence father for next generation (*third generation*)

As this manner the first (changed sons to fathers) father of second generation after doing {R.G.G}, he will do

- 1- as a gift he puts the greatest coefficients of his own equation as the last coefficients for each one of the terms (he increase the numbers of coefficients of terms)
- 2- as another gift he transfer his equation completely to next generation (without eliminating the last term)

The obtained equation is the first son of first father from second generation for third generation



The first father by using of the first son's sample and on the basis of below rules generates the set of sons with a same last coefficients of terms (as identity code for equation)

- 1- Eliminate one unit of the last terms of equation step by step of generating the sons
- 2- Reduce one unit of each base numbers located in numerator of parenthesis in terms step by step of generating the sons
- 3- reduces one unit of value the coefficients locate between the first and the last coefficients step by step of generating the sons (the first and the last coefficients are invariable and fixed figures)

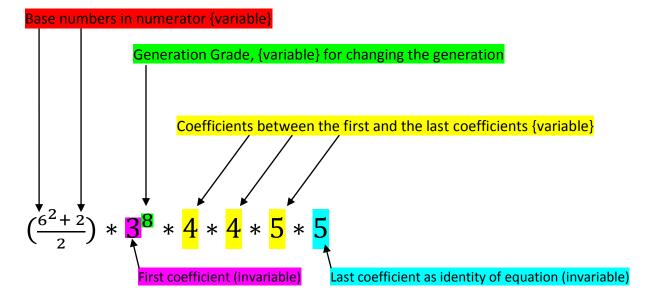
For the fallowing example Human's package $\{7770\}$ located in $\{n = 9\}$ and $\{k = 4\}$ the set of the sons (*brothers*) are as down figure



Then the second father of previous generation (second generation) after doing {R.G.G}

- 1- as a gift he puts the greatest coefficients of his own equation as the last coefficients for each one of the terms (he increase the numbers of coefficients of terms)
- 2- as another gift he transfer his equation completely to next generation (without eliminating the last term)

The obtained equation is the second son of second father from second generation then the father on the basis the sample of generated son and by using of related rules will create his son's set (below, sample of a term)





The third father by using of the first son's sample and on the basis of below rules generates the set of sons with a Same last coefficients of terms as identity code

- 1- Eliminate one unit of the last terms of equation step by step of generated sons
- 2- reduce one unit of each base numbers located in numerator of parenthesis in terms step by step of generated sons
- 3- reduces one unit of the coefficients located between the first and the last coefficients step by step of generating the sons (the first and the last coefficients are invariable and fixed figures)

For the fallowing example human's package $\{7770\}$ located in $\{n = 9\}$ and $\{k = 4\}$ the set of the sons (*brothers*) are as down figure (*below is a sample of sons set for showing the variable and invariable values in creating the sons set*)

$$\left(\frac{3^{2}+1}{2}\right) * \mathbf{1}^{4} * \mathbf{3} * \mathbf{3} + \left(\frac{3^{2}-1}{2}\right) * \mathbf{2}^{4} * \mathbf{3} * \mathbf{3} = 621$$

 $\left(\frac{2^{2}-0}{2}\right) * \mathbf{1}^{4} * \mathbf{2} * \mathbf{3} = 12$



Last generation and the last equation of Humans packages:

As we can see after doing the $\{R.G.G\}$ for next generation the Generation Grade $\{G.G\}$ in equations will be $\{1\}$ and the equations with $\{G.G\}$ smaller than $\{2\}$ is not humans equation Then for the fallowing example human's package $\{7770\}$ located in $\{n = 9\}$ and $\{k = 4\}$ the third generation will be the last generation

Important general rule:

As a general rule the generating rules for Human's package from **third generating section till the last generating** section are **repetitive** rules and by using of one packages rules easily we can create the other Stirling numbers packages

Then by putting the set of stirling packages which are locate in a same row of a stirling triangular array we can study about stirling and bell and lah and other related subjects to Stirling numbers and also the fractal issue

for example the set of stirling Human's packages located in row No. {9} are as down figure



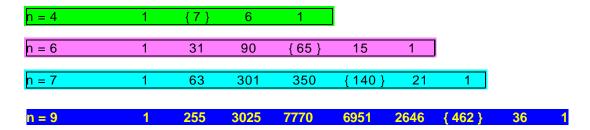
Jinni's Stirling numbers packages

Jinni section:

In every one of the Stirling number's package locates in a same row of a stirling triangular array there is a separate section for jinni's generation

The jinni's common prim fathers are locate in third numbers from right of the stirling set

For example the common jinni's prim fathers for rows No. (n = 4) & (n = 6) & (n = 7) & (n = 9) are locate in stirling numbers (7) & (65) & (140) & (462), as down figure.



Here we are going to describe the process of creating the jinni's section in each one of the related package for Stirling numbers set via an example for row No. (n = 9) and related jinni's prim father in Stirling number $\{462\}$



As we know the structure of jinni is plainer than the Human but there are some similarity among Humans and Jinni in condition of generating the children

for example the equation of the Human's father for stirling number {7770} is as down figure

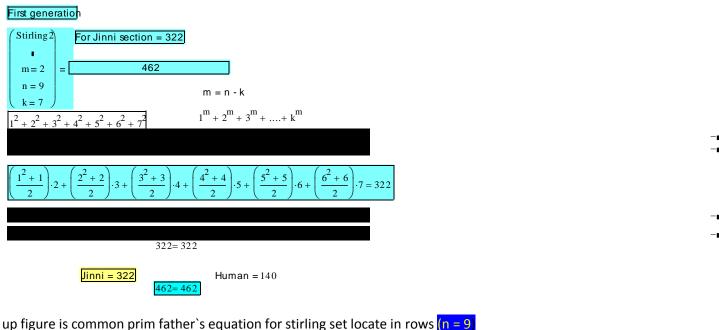
$$\left[\frac{4^2 + 2}{2} \right) \cdot 1^4 + \left(\frac{4^2 + 0}{2} \right) \cdot 2^4 + \left(\frac{4^2 - 2}{2} \right) \cdot 3^4 + \left(\frac{4^2 - 4}{2} \right) \cdot 4^4 = 2240$$
 or
$$\left[(2 + 3 + 4) \cdot 1^4 + (1 + 3 + 4) \cdot 2^4 + (1 + 2 + 4) \cdot 3^4 + (1 + 2 + 3) \cdot 4^4 = 2240 \right]$$

And the equation of common prim father for stirling number set locate in row No. {9} in jinni section is as down figure

$$\left(\frac{1^2+1}{2}\right) \cdot 2 + \left(\frac{2^2+2}{2}\right) \cdot 3 + \left(\frac{3^2+3}{2}\right) \cdot 4 + \left(\frac{4^2+4}{2}\right) \cdot 5 + \left(\frac{5^2+5}{2}\right) \cdot 6 + \left(\frac{6^2+6}{2}\right) \cdot 7 = 322$$

As we can see the exponent (*generation grade*) of the first coefficients for parenthesis in jinni's equations are always only one

Down figure is common prim father's equation for stirling set locate in rows (n = 9)



up figure is common prim father's equation for stirling set locate in rows [11=9]

n = 9 1 255 3025 7770 6951 2646 { 462 } 36

The jinn's prim fathers (like Humans prim fathers) have two opposite characters

- 1- **Good and benevolent father**: as a gift he puts the greatest coefficients of his equation as the last coefficients for each one of the terms (he increase the numbers of coefficients for each one of the terms)
- 2- **Bad father for future generation**: as jealousy he eliminate the greatest term of his equation from right end of equation

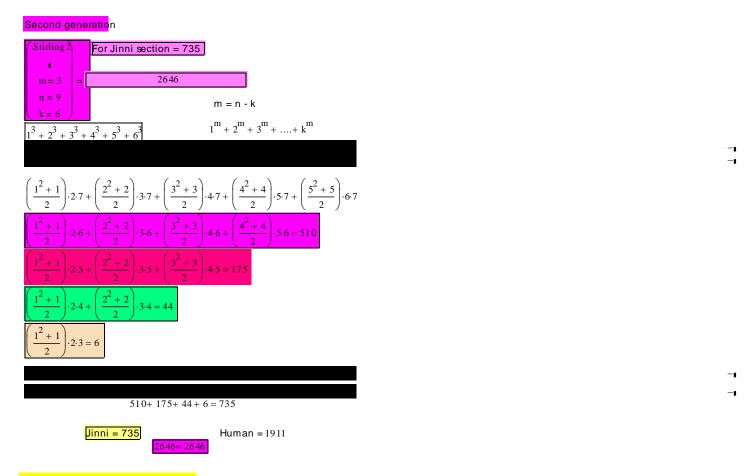
The generated equation is the first son (*first born*) in second generation (*for above example No.* (n = 9) and (2646)) is as down figure

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 7 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 7 + \left(\frac{3^2+3}{2}\right) \cdot 4 \cdot 7 + \left(\frac{4^2+4}{2}\right) \cdot 5 \cdot 7 + \left(\frac{5^2+5}{2}\right) \cdot 6 \cdot 7 = 1225$$

The common jinni's prim father on the basis of the first son's sample and using of below rules generates the first set of sons in second generation

- 1- Eliminate one by one the last terms of sons equations step by step of generating the sons
- 2- reduces one unit of the last coefficients of terms (the first coefficients are invariable and fixed figures) step by step of generating the sons

Set of the sons in second generation (for above example No (n = 9) and (2646)) are as down figure



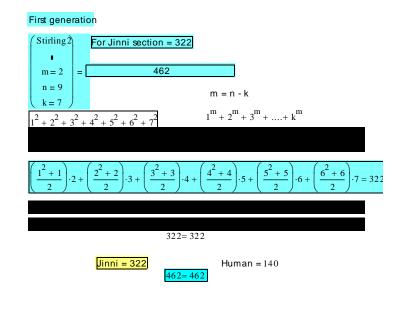
Main jinni section rule: (E.E.B.K) or (Eliminating Equations Bigger than K)

After generating the jinni sons in one related package the sons (*equations*) with the last coefficients (*as identity code for equation*) bigger than {K}, (k = *column number*) should be eliminate (*undefined equations for recent package*) of recent package It means that in jinni's related packages only the equations (*jinni's sons*) with the last coefficients (*as identity code for equation*) equal or smaller than {K}, (k = *column number*) have right to be

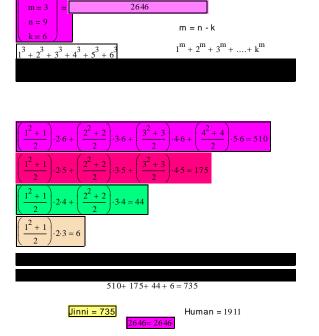
For example: the below equation (jinni son) has no permission to be in jinni's package $\{2646\}$ with coordinate $\{k = 6\}$ and $\{n = 9\}$ because the last coefficients of his terms are $\{7\}$ and it is bigger than $\{k = 6\}$

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 7 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 7 + \left(\frac{3^2+3}{2}\right) \cdot 4 \cdot 7 + \left(\frac{4^2+4}{2}\right) \cdot 5 \cdot 7 + \left(\frac{5^2+5}{2}\right) \cdot 6 \cdot 7 = 1225$$

Second generation Stirling 2 m = 3 n = 9 k = 6 $1^{3} + 2^{3} + 3^{3} + 4^{3} + 5^{3} + 6^{3}$ $\frac{1^{3} + 2^{3} + 3^{3} + 4^{3} + 5^{3} + 6^{3}}{2}$ $\frac{1^{2} + 1}{2} \cdot 2 \cdot 7 + \left(\frac{2^{2} + 2}{2}\right) \cdot 3 \cdot 7 + \left(\frac{3^{2} + 3}{2}\right) \cdot 4 \cdot 7 + \left(\frac{4^{2} + 4}{2}\right) \cdot 5 \cdot 7 + \left(\frac{5^{2} + 5}{2}\right) \cdot 6 \cdot 7$ $\frac{1^{2} + 1}{2} \cdot 2 \cdot 6 + \left(\frac{2^{2} + 2}{2}\right) \cdot 3 \cdot 6 + \left(\frac{3^{2} + 3}{2}\right) \cdot 4 \cdot 6 + \left(\frac{4^{2} + 4}{2}\right) \cdot 5 \cdot 6 = 510$ $\frac{1^{2} + 1}{2} \cdot 2 \cdot 5 + \left(\frac{2^{2} + 2}{2}\right) \cdot 3 \cdot 5 + \left(\frac{3^{2} + 3}{2}\right) \cdot 4 \cdot 5 = 175$ $\frac{1^{2} + 1}{2} \cdot 2 \cdot 4 + \left(\frac{2^{2} + 2}{2}\right) \cdot 3 \cdot 4 = 44$ $\frac{1^{2} + 1}{2} \cdot 2 \cdot 3 = 6$ Human = 1911

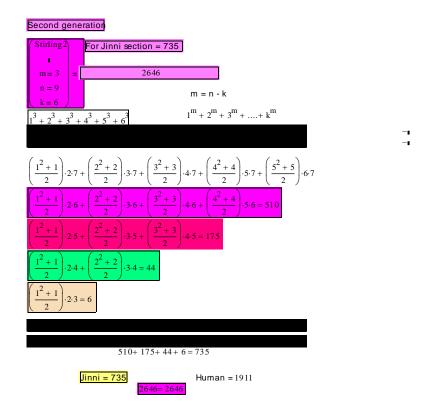


The up mentioned package $\{2646\}$ after (E.E.B.K) Eliminate the Equation with the last coefficients (as identity code) Bigger than $\{k\}$, $\{k = 6\}$ is as down figure



Second generation

For Jinni section = 735



After completion the generating all sons (*jinni*'s sons which generated by prim father) in second generation, **each one of** the generated sons will change to a good and benevolence father for next generation (third generation) as this manner the first changed son to a good father (in second generation) will do

- 1- For a gift he puts the greatest coefficients of his equation as the last coefficients for each one of the terms (he increase the numbers of coefficients of terms in his equation)
- 2- for another gift he transfer his equation completely to next generation (without eliminating the last term)

The generated son will be locate in package $\{6951\}$ with coordinates $\{k = 5\}$ and $\{n = 9\}$

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 6 \cdot 6 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 6 \cdot 6 + \left(\frac{3^2+3}{2}\right) \cdot 4 \cdot 6 \cdot 6 + \left(\frac{4^2+4}{2}\right) \cdot 5 \cdot 6 \cdot 6 = 3060$$

According to Main jinni section's rule (E.E.B.K), the above generated son and the other brothers which will generate via him should be eliminate from recent package $\{6951\}$ with coordinates $\{k = 5\}$ and $\{n = 9\}$ because the last coefficients (as identity code) of terms are $\{6\}$ and they are bigger than $\{k = 5\}$

Therefore the second father in second generation will generate his son for third generation under below rules

- 1- For a gift he puts the greatest coefficients of his equation as the last coefficients for each one of the terms (he increase the numbers of coefficients of terms)
- 2- for another gift he transfer his equation completely to next generation (without eliminating the last term)

The generated son will be locate in package of $\{6951\}$ with coordinates $\{k = 5\}$ and $\{n = 9\}$ as first son in third generation as down figure

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 5 \cdot 5 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 5 \cdot 5 + \left(\frac{3^2+3}{2}\right) \cdot 4 \cdot 5 \cdot 5 = 875$$

Then the first father in second generation with the last coefficients {5}, on the basis of the first son`s sample and using of below rules generates the **first set of sons** in third generation

- 1- Eliminate one by one the last terms of sons equations step by step of generating the sons
- 2- reduces one unit of the last coefficients of terms (the first and the last coefficients are invariable and fixed figures) step by step of generating the sons

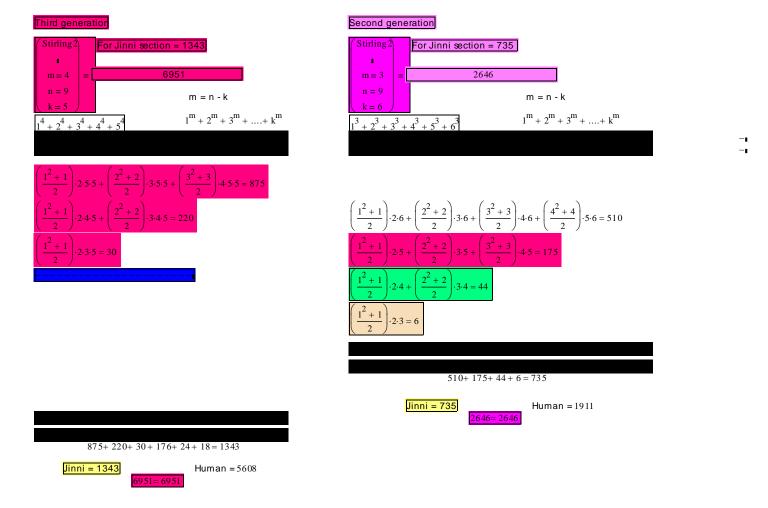
The first set of sons in third generation, package of {6951} is as down figure

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 5 \cdot 5 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 5 \cdot 5 + \left(\frac{3^2+3}{2}\right) \cdot 4 \cdot 5 \cdot 5 = 875$$

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 4 \cdot 5 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 4 \cdot 5 = 220$$

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 3 \cdot 5 = 30$$

And also the complete diagram of above sons set is as down figure



Then the second father with identity {4}, in second generation will start to generate his first son for next generation

- 1- For a gift he puts the greatest coefficients of his equation (*identity value of equation*) as the last coefficients for each one of the terms (*he increase the numbers of coefficients of terms*)
- 2- For another gift he transfer the obtained equation completely to next generation (*without eliminating the last term of equation*)

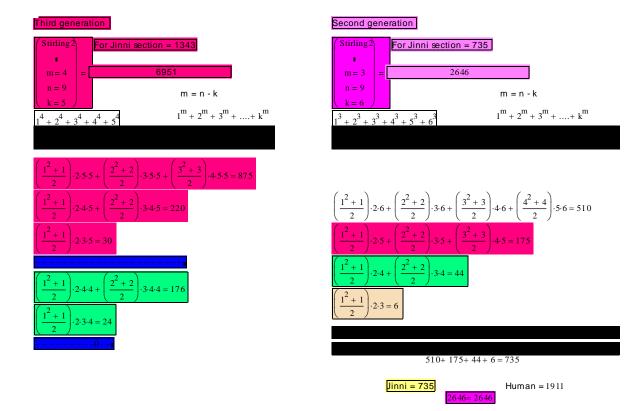
The generated son will be locate in package $\{6951\}$ with coordinates $\{k = 5\}$ and $\{n = 9\}$ as first son with identity code $\{4\}$

$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 4 \cdot 4 + \left(\frac{2^2+2}{2}\right) \cdot 3 \cdot 4 \cdot 4 = 176$$

Then the mentioned father (with identity {4}) on the basis of the first son's sample and using of below rules generates the second set of sons (with identity {4}) in third generation

- 1- Eliminate the last terms of equation step by step in generating the sons
- 2- reduces one unit of the coefficients locate between the first and the last coefficients of terms (the first and the last coefficients are invariable and fixed figures) step by step in generating the sons

The second set of sons (with identity {4}) in third generation package {6951} is as down figure



Then the third father in second generation with identity {3} (the last father in second generation)

- 1- For a gift he puts the greatest coefficients of his equation (*identity value of equation*) as the last coefficients for each one of the terms (*he increase the numbers of coefficients of terms*)
- 2- For another gift he transfer the obtained equation completely to next generation (*without eliminating the last term of equation*)

The generated son will be locate in package $\{6951\}$ with coordinates $\{k = 5\}$ and $\{n = 9\}$ as first son with identity code $\{3\}$

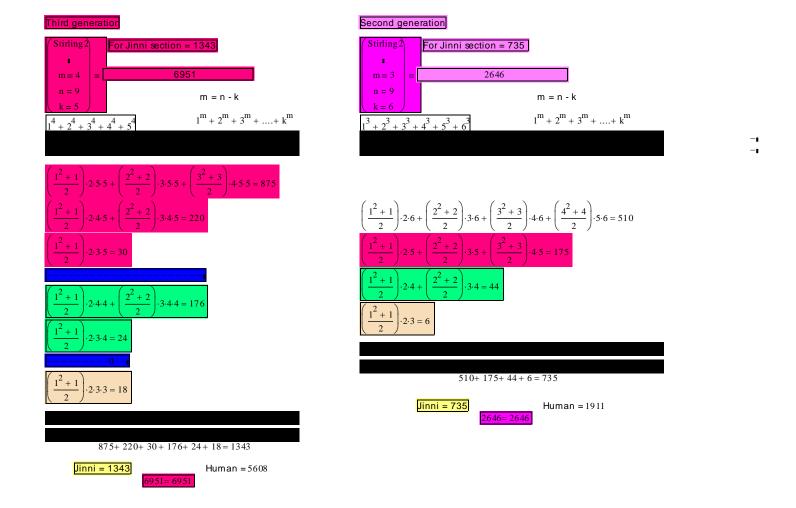
$$\left(\frac{1^2+1}{2}\right) \cdot 2 \cdot 3 \cdot 3 = 18$$

Then the mentioned father (with identity {3}) on the basis of the first son's sample and using of below rules generates the **third set of sons** (with identity {3}) in third generation

- 1- Eliminate the last terms of equation step by step in generating the sons
- 2- reduces one unit of the coefficients locate between the first and the last coefficients of terms (the first and the last coefficients are invariable and fixed figures) step by step in generating the sons

The third set of sons (with identity {3}) in third generation package {6951} is as down figure

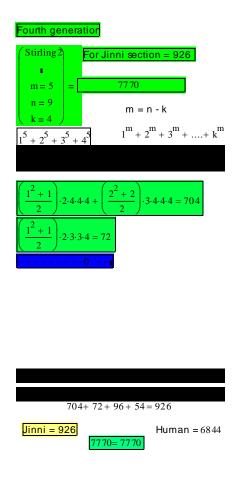
As we can see for fallowing example the third set (with identity {3}) of sons in third generation is only one single son

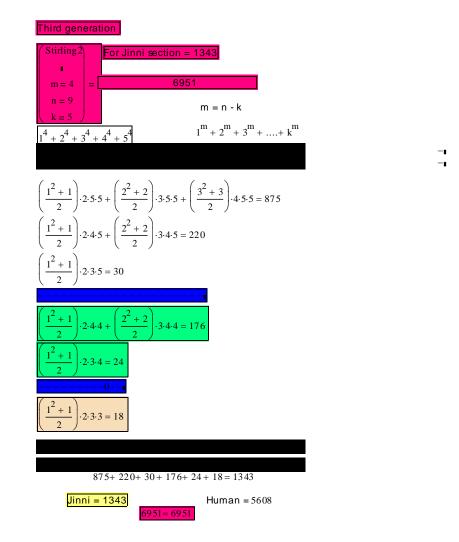


After completion the generating all sons (*jinni*'s sons) in third generation, **each one of the generated sons will change to a good and benevolence father for next generation** (*fourth generation*) as this manner the first changed son to a good father in third generation

According to **Main jinni section's rule** (**E.E.B.K**), the generated sons of first fathers set and the other brothers which will generate via them (with identity code $\{5\}$) should be eliminate from recent package stirling $\{7770\}$ with coordinates $\{k = 4\}$ and $\{n = 9\}$ because the last coefficients (as identity code) of terms are $\{5\}$ and they are bigger than $\{k = 4\}$

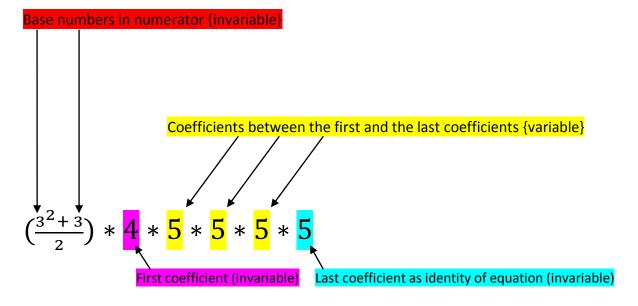
Then the forth father of third generation (the first father of second set with identity $\{4\}$) will generate his first son and on the basis of first son's sample and by using of **G**enerating the **S**ons **S**et's rule (**G.S.S**) he will generate the first set of sons (with identity $\{4\}$) in fourth generation in package $\{7770\}$ & $\{n = 9\}$ & $\{k = 4\}$

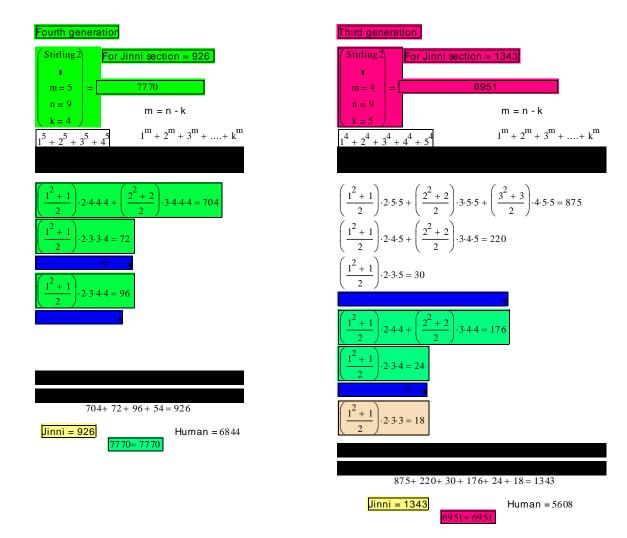




Then the fifth father of third generation (the first father of second set with identity $\{4\}$) will generate his first son and on the basis of first son's sample and by using of Generating the Sons Set's rule (G.S.S) he will generate the second set of sons (with identity $\{4\}$), in fourth generation in package $\{7770\}$ & $\{n = 9\}$ & $\{k = 4\}$ } here in fallowing example the generated son's set (identity $\{4\}$) in this stage is only one single son

Below is a sample of a term in jinni's equation





Then the sixth father of third generation (the first father of third set with identity $\{3\}$) will generate his first son and on the basis of first son's sample and by using of **G**enerating the **S**ons **S**et's rule **(G.S.S)** he will generate the third set of sons (with identity $\{3\}$) in fourth generation in package $\{7770\}$ & $\{n = 9\}$ & $\{k = 4\}$

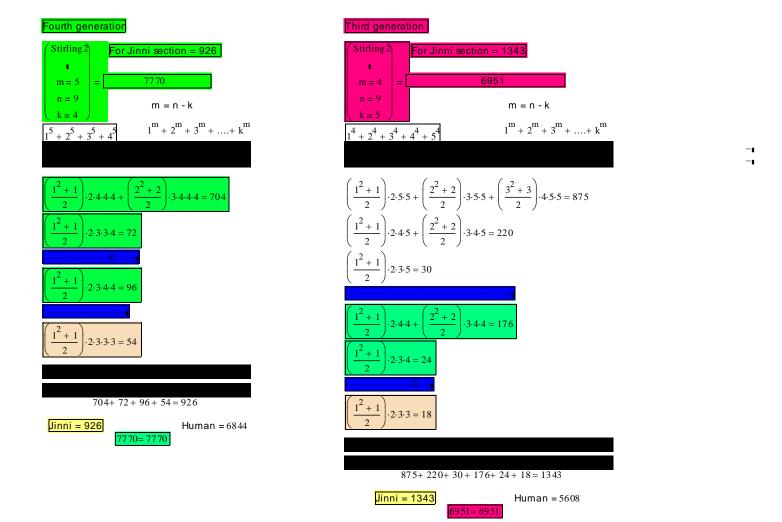
Here in fallowing example the generated son's set (identity {3}) in this stage is only one single son

An example about jinni's sons set is as down equations

$$(\frac{1^2+1}{2}) * \mathbf{2} * \mathbf{5} * \mathbf{5} * \mathbf{5} * \mathbf{5} + (\frac{2^2+2}{2}) * \mathbf{3} * \mathbf{5} * \mathbf{5} * \mathbf{5} + (\frac{3^2+3}{2}) * \mathbf{4} * \mathbf{5} * \mathbf{5} * \mathbf{5} = 4375$$

$$(\frac{1^2+1}{2}) * \mathbf{2} * \mathbf{4} * \mathbf{4} * \mathbf{5} + (\frac{2^2+2}{2}) * \mathbf{3} * \mathbf{4} * \mathbf{4} * \mathbf{5} = 880$$

$$(\frac{1^2+1}{2}) * \mathbf{2} * \mathbf{3} * \mathbf{3} * \mathbf{5} = 90$$



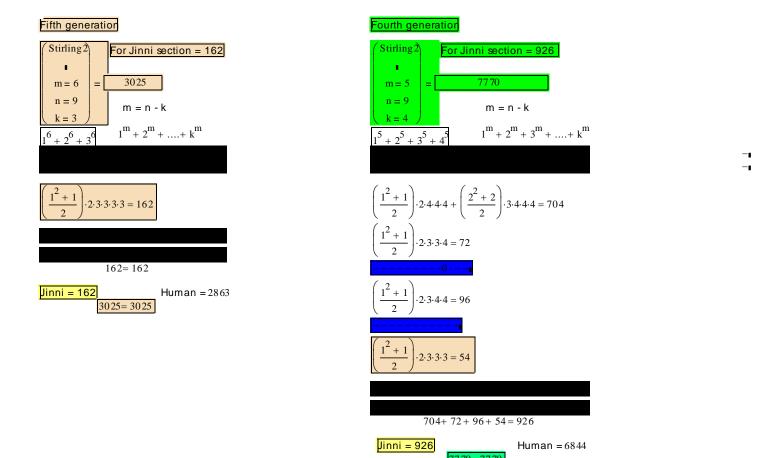
After completion the generating all brothers (*jinni*'s sons) in fourth generation, **each one of the generated brothers** will change to a good and benevolence father for next generation (*fifth generation*) as this manner the first changed son to a good father in third generation

According to **Main jinni section's** rule (**E.E.B.K**), the generated sons of fathers set and the other brothers which will generate via them (with identity code $\{4\}$) should be eliminate from recent package $\{3025\}$ with coordinates $\{k = 3\}$ and $\{n = 9\}$ because the last coefficients (identity code) of terms are $\{4\}$ and they are bigger than $\{k = 3\}$

Then the fourth father of fourth generation (the first father of fourth set with identity $\{3\}$) will generate his first son and on the basis of first son's sample and by using of **generating the Sons Set**'s rule **(G.S.S)** he will generate the first set of sons (with identity $\{3\}$) in fifth generation in package $\{3025\}$ & $\{n = 9\}$ & $\{k = 3\}$

Important general rule:

As a general rule the generating rules for jinni's packages (as Human packages) from third generating section till the last generating section are repetitive rules and by using of one packages rules easily we can create the other Stirling numbers Jinni packages

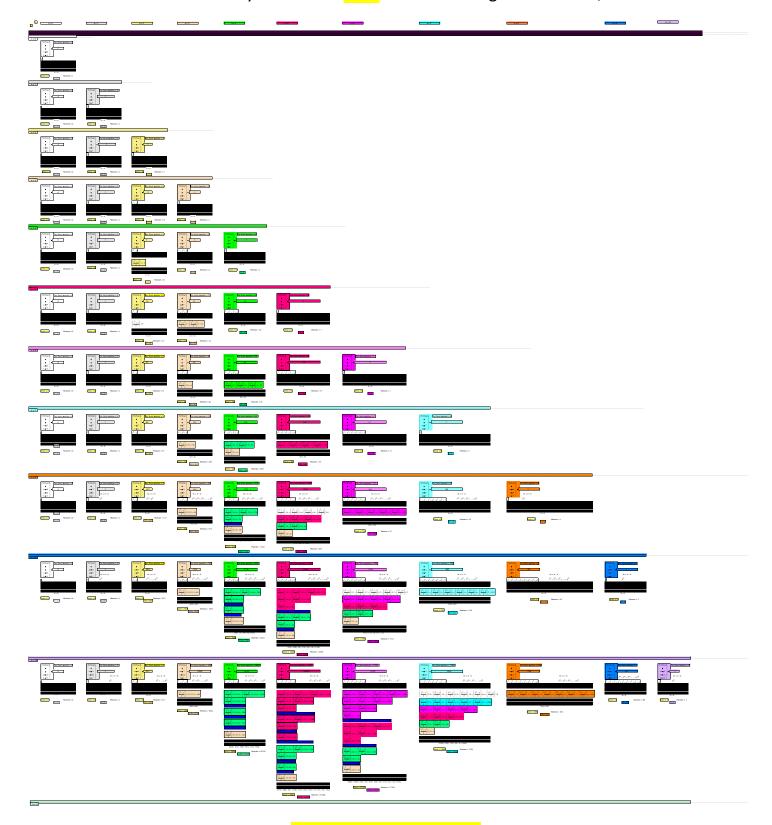


Therefore in fallowing example the package {3025} is the last jinni package and the next of it {255} is empty of jinni son Below is a complete set of jinni's packages for Stirling number's set locate in row No.{9}



Stirling numbers of the second kind triangular set of equation packages array

Based on family tree of the Jinni and Human algorithm Serajian Asl



End of article

Stirling Magic Cube (Serajian Asl)

"Figure 1"

5								
$n \setminus k$	1	2	3	4	5	6	7	8
1	1							
2	1	1						
3	1	3	1					
4	1	7	6	1				
5	1	15	25	10	1			
6	1	31	90	65	15	1		
7	1	63	301	350	140	21	1	
8	1	127	966	1701	1050	266	28	1
9	1	255	3025	7770	6951	2646	462	36
10	1							

By transferring the Stirling triangular array's columns to top of the array, we will have a **squared Array** named **"Stirling numerical squared array"**

"Figure 2"

1154								
$n \setminus k$	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	3	6	10	15	21	28	36
3	1	7	25	65	140	266	462	750
4	1	15	90	350	1050	2646	5880	11880
5	1	31	301	1701	6951	22827	63987	159027
6	1	63	966	7770	42525	179487	627396	1899612
7	1	127	3025	34105	246730	1323652	5715424	20912320
8	1	255	9330	145750	1379400	9321312	49329280	216627840
9	1							

Each one of the Stirling numbers that locates in rows of **Stirling numerical squared array**, is Equal with a **math relation**

For example series of the numbers which locates in row n=3 of Stirling numerical squared array is as below

And each one of the numbers in above series is equal with a equation

Ingule 3

....1 =
$$1 \cdot \left(\frac{1^3 + 1^2}{2}\right)$$

....7 = $1 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 1 \cdot \left(\frac{2^3 + 2^2}{2}\right)$

...25 = $1 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 1 \cdot \left(\frac{2^3 + 2^2}{2}\right) + 1 \cdot \left(\frac{3^3 + 3^2}{2}\right)$

...65 = $1 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 1 \cdot \left(\frac{2^3 + 2^2}{2}\right) + 1 \cdot \left(\frac{3^3 + 3^2}{2}\right) + 1 \cdot \left(\frac{4^3 + 4^2}{2}\right)$

$$140 = 1 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 1 \cdot \left(\frac{2^3 + 2^2}{2}\right) + 1 \cdot \left(\frac{3^3 + 3^2}{2}\right) + 1 \cdot \left(\frac{4^3 + 4^2}{2}\right) + 1 \cdot \left(\frac{5^3 + 5^2}{2}\right)$$

The set of **parenthesis's coefficients** in up relations make a triangular array as below "Figure 4"

1		_					
1	1		_				
1	1	1		_			
1	1	1	1		-		
1	1	1	1	1		_	
1	1	1	1	1	1		-
1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1

The other example, is set of numbers which locates in row n = 4 of Stirling numerical squared array.

And each one of the numbers in above series is equal with a math relation "Figure 5"

$$.....15 = ...3 \cdot \left(\frac{1^3 + 1^2}{2}\right) + ...2 \cdot \left(\frac{2^3 + 2^2}{2}\right) \\
.....90 = ...6 \cdot \left(\frac{1^3 + 1^2}{2}\right) + ...5 \cdot \left(\frac{2^3 + 2^2}{2}\right) + ...3 \cdot \left(\frac{3^3 + 3^2}{2}\right) \\
....350 = 10 \cdot \left(\frac{1^3 + 1^2}{2}\right) + ...9 \cdot \left(\frac{2^3 + 2^2}{2}\right) + ...7 \cdot \left(\frac{3^3 + 3^2}{2}\right) + ...4 \cdot \left(\frac{4^3 + 4^2}{2}\right) \\
1050 = 15 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 14 \cdot \left(\frac{2^3 + 2^2}{2}\right) + 12 \cdot \left(\frac{3^3 + 3^2}{2}\right) + ...9 \cdot \left(\frac{4^3 + 4^2}{2}\right) + ...5 \cdot \left(\frac{5^3 + 5^2}{2}\right) \\
2646 = 21 \cdot \left(\frac{1^3 + 1^2}{2}\right) + 20 \cdot \left(\frac{2^3 + 2^2}{2}\right) + 18 \cdot \left(\frac{3^3 + 3^2}{2}\right) + 15 \cdot \left(\frac{4^3 + 4^2}{2}\right) + 11 \cdot \left(\frac{5^3 + 5^2}{2}\right) + 6 \cdot \left(\frac{6^3 + 6^2}{2}\right)$$

Set of the **parenthesis's coefficients** in up relations make a triangular array as below "Figure 6"

Squared array No.3 created on the basis of row n=3 in Stirling squared array

"Figure 7"

Set of the **parenthesis's coefficients** in up relations make a triangular array as below

"Figure 6"

1							
3	2		-				
6	5	3		F			
10	9	7	4		_		
15	14	12	9	5		_	
21	20	18	15	11	6		_
28	27	25	22	18	13	7	
36	35	33	30	26	21	15	8

By transferring the columns of above made triangular arrays, to top of arrays we will create

squared

$n \setminus k$	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1
9	1							

Squared array No.4 created on the basis of row n=4 in Stirling squared array

"Figure 8"

0								
$n \setminus k$	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	3	5	7	9	11	13	15	17
3	6	9	12	15	18	21	24	27
4	10	14	18	22	26	30	34	38
5	15	20	25	30	35	40	45	50
6	21	27	33	39	45	51	57	63
7	28	35	42	49	56	63	70	77
8	36	44	52	60	68	76	84	92
9	45							

Squared array No.5 created on the basis of row n=5 in Stirling squared array

"Figure 9"

1154	10)							
$n \setminus k$	1	2	3	4	5	6	7	8
1	1	4	9	16	25	36	49	64
2	7	19	37	61	91	127	169	217
3	25	55	97	151	217	295	385	487
4	65	125	205	305	425	565	725	905
5	140	245	380	545	740	965	1220	1505
6	266	434	644	896	1190	1526	1904	2324
7	462	714	1022	1386	1806	2282	2814	3402
8	750	1110	1542	2046	2622	3270	3990	4782
	1155							

Squared array No.6 created on the basis of row n=6 in Stirling squared array

"Figure 10"

$n \setminus k$	1	2	3	4	5	6	7	8
1	1	8	27	64	125	216	343	512
2	15	65	175	369	671	1105	1695	2465
3	90	285	660	1275	2190	3465	5160	7335
4	350	910	1890	3410	5590	8550	12410	17290
5	1050	2380	4550	7770	12250	18200	25830	35350
6	2646	5418	9702	15834	24150	34986	48678	65562
7	5880	11130	18900	29694	44016	62370	85260	113190
8	11880	21120	34320	52200	75480	104880	141120	184920
9	22275							

Squared array No.7 created on the basis of row n=7 in Stirling squared array "need to be complete"

"Figure 11"

$n \setminus k$	1	2	3	4	5	6	7	8
1	1	16	81	256	625	1296	2401	4096
2	31	211	781	2101	4651	9031	15961	26281
3	301	1351	4081	9751	19981	36751	62401	
4	1701	5901	15421	33621	64701	113701		
5	6951	20181	47271	95781	174951			
6	22827	58107	124887	238287				
7	63987	147147	294987					
8	159027	337227						
9	359502							

Squared array No.8 created on the basis of row *n*=8 in Stirling squared array "need to be complete" "Figure 12"

$n \setminus k$	1	2	3	4	5	6	7	8
1	1	32	243	1024	3125	7776	16807	32768
2	63	665	3367	11529	31031	70993	144495	02.00
3	966	6069	23772	70035	170898	365001		
4	7770	35574	116298	305382	688506			
5	42525	156660	447195	1071630				
6	179487	563409	1446291					
7	627396	1740585						
8	1899612							
9								

Set of the above made squared arrays makes a three – dimensional "3D numerical array" In the name **numerical cube array**

Important points:

In all of obtained squared arrays, the values which locate in first columns " k = 1" are as same as the numbers which locate in a row of squared Stirling array

In all of obtained squared arrays, the values which locate in first rows " n = 1" are the powers of first natural numbers.

The numbers are located in same **columns** or **rows** or **diagonals** on each one of the squared array, have relations with together and make progression series in different deeps. For example progression series of numbers which locate in row "n = 3" of squared array "a = 5" Is as below

By changing the number of "n or k" in squared arrays the deep of sequences will change

For example sequence of values locate in row n=3 of squared array a=6 is as below.

90		285		660		1275	:	2190	3	3465		5160		7335
	195		375		615		915		1275		1695		2175	
		180		240		300		360		420		480)	
			60		60		60		60		60			

Also each one of the squared array have relations with **previous** or **next** squared arrays by two below relations or formulas and the numbers of them make chain stitch **sequence** with together

$$\begin{pmatrix} a \\ n \\ k \end{pmatrix} = \begin{pmatrix} a \\ n-1 \\ k \end{pmatrix} + \begin{pmatrix} a-1 \\ n \\ k \end{pmatrix} \cdot (n+k-1)$$

Example for relation 1

array
$$a=6$$
 row $n=7$ column $k=3$

$$\begin{pmatrix} 6 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 - 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 - 1 \\ 7 \\ 3 \end{pmatrix} \cdot (7 + 3 - 1)$$

$$(18900) = (9702) + (1022) \cdot (7 + 3 - 1)$$

$$\begin{pmatrix} a \\ n \\ k \end{pmatrix} = \begin{pmatrix} a \\ n-1 \\ k+1 \end{pmatrix} + \begin{pmatrix} a-1 \\ n \\ k \end{pmatrix} \cdot (k)$$

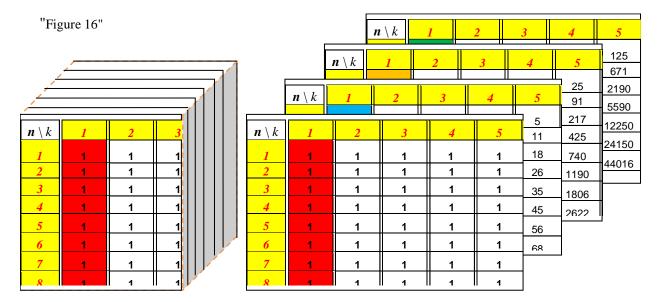
Example for relation 2

array a=6 row n=7 column k=3

$$\begin{pmatrix} 6 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 - 1 \\ 3 + 1 \end{pmatrix} + \begin{pmatrix} 6 - 1 \\ 7 \\ 3 \end{pmatrix} \cdot (k)$$

$$(18900) = (15834) + (1022) \cdot (3)$$

Set of the above made squared arrays makes a three – dimensional "3D numerical array" In the name **numerical cube array**



Numerical cube set of the squared arrays make three dimensional numerical cube array

By adding {3} to each one of base numbers, the 4th term of sequence will be obtain {42525, 156660,447195, 1071630, ...}

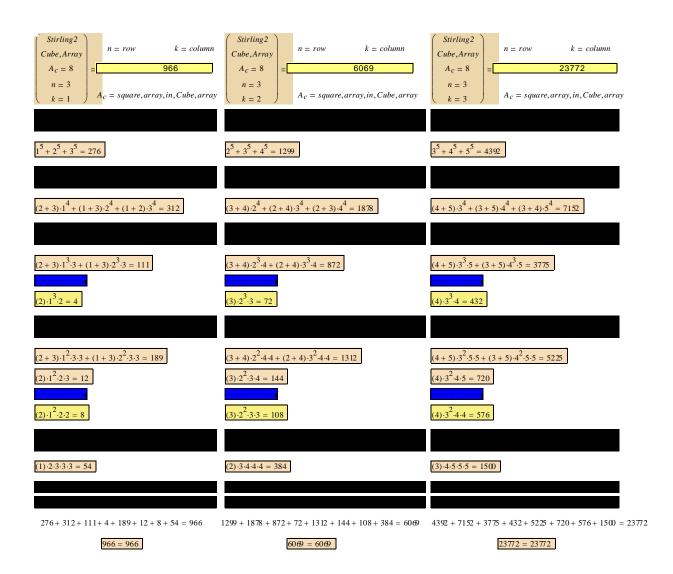




How to create the equation package for other cube array numbers

For creating the other cube array numbers this will be enough that we add "N" unit To base of all numbers in a Stirling package "the power numbers have no any of changes" it means that, for example by adding one unit to all basis of exponents numbers in Stirling package {966} we will obtain the equation of "6069" and by adding two unit to all basis of exponents numbers in Stirling package {966} we will obtain Stirling package {23772} and so (it can be a Fractal)

"Figure 47"



Down sequences locates in $\{\text{squared array} = 8\}$ and $\{\text{n} = 5\}$ and $\{\text{k} = 4\}$ is a fifth difference sequence which has obtained by adding the values $\{1 \text{ till } 8\}$ to all base numbers in Stirling number package $\{42525\}$

{42525, 156660, 447195, <mark>1071630, ...</mark>}

42525		156660		447195		1071630		2263065		4345320		7748055		13021890		20853525	
	114135		290535		624435		1191435		2082255		3402735		5273835		7831635		11227335
		176400		333900		567000		890820		1320480		1871100		2557800		3395700	
			157500		233100		323820		429660		550620		686700		837900		1004220
				75600		90720		105840		120960		136080		151200		166320	
					15120		15120		15120		15120		15120		15120		15120

The Stirling magic cube {serajian} is a really magic cube because all line of it as rows, columns, diagonals, and other lines as dimensional lines in a numerical cube make series of nth differences sequences

This kind of magic cube is considerable and need for some research on it down is some sequences of Stirling magic cube

Squared array No.6 row No $\{n = 6\}$

2646		5418		9702		15834		24150		34986
	2772		4284		6132		8316		10836	
		1512		1848		2184		2520		2856
			336		336		336		336	

Squared array No.6 diagonal No.1 row No. {n = 1,2,3,4,...}

1		65		660		3410		12250		34986		85260		184920		366795
	64		595		2750		8840		22736		50274		99660		181875	
		531		2155		6090		13896		27538		49386		82215		
			1624		3935		7806		13642		21848		32829			
				2311		3871		5836		8206		10981				
					1560		1965		2370		2775					
						405		405		405						

Squared array No.6 column No. {k = 1}

1		15		90		350		1050		2646		5880		11880
	14		75		260		700		1596		3234		6000	
		61		185		440		896		1638		2766		
			124		255		456		742		1128			
				131		201		286		386				
					70		85		100					
						15		15						

Equations of packages {42525} for adding values {1,2,3,..N} instead of {A1} in Excel for getting the term of sequence

		0 0
1+A1)^5+(2+A1)^5+(3+A1)^5+(4+A1)^5+(5+A1)^5		4425
2+3+4+5+4*A1)*(1+A1)^4+(1+3+4+5+4*A1)*(2+A1)^4+(1+2+4+5+4*A1)*(3+A1)^4+(1+2+3+5+4*A1)*(4+A1)^4+(1+2+3+4+4*A1)*(5+A1)^4		1026
2+3+4+5+4*A1)*(1+A1)^3*(5+A1)+(1+3+4+5+4*A1)*(2+A1)^3*(5+A1)+(1+2+4+5+4*A1)*(3+A1)^3*(5+A1)+(1+2+3+5+4*A1)*(4+A1)^3*(5+A1)		5730
2+3+4+3*A1)*(1+A1)^3*(4+A1)+(1+3+4+3*A1)*(2+A1)^3*(4+A1)+(1+2+4+3*A1)*(3+A1)^3*(4+A1)		1048
2+3+2*A1)*(1+A1)^3*(3+A1)+(1+3+2*A1)*(2+A1)^3*(3+A1)		111
2+1*A1)*(1+A1)^3*(2+A1)		4
2+3+4+5+4*A1)*(1+A1)^2*(5+A1)*(5+A1)+(1+3+4+5+4*A1)*(2+A1)^2*(5+A1)+(1+2+4+5+4*A1)*(3+A1)^2*(5+A1)*(5+A1)+(1+2+3+5+4*A1)*(4+A1)	^2*(5+A1)*(5+A1)	8750
2+3+4+3*A1)*(1+A1)^2*(4+A1)*(5+A1)+(1+3+4+3*A1)*(2+A1)^2*(4+A1)*(5+A1)+(1+2+4+3*A1)*(3+A1)^2*(4+A1)*(5+A1)		2080
2+3+2*A1)*(1+A1)^2*(3+A1)*(5+A1)+(1+3+2*A1)*(2+A1)^2*(3+A1)*(5+A1)		315
2+1*A1)*(1+A1)^2*(2+A1)*(5+A1)		20
2+3+4+3*A1)*(1+A1)^2*(4+A1)*(4+A1)+(1+3+4+3*A1)*(2+A1)^2*(4+A1)*(4+A1)+(1+2+4+3*A1)*(3+A1)^2*(4+A1)*(4+A1)		1664
2+3+2*A1)*(1+A1)^2*(3+A1)*(4+A1)+(1+3+2*A1)*(2+A1)^2*(3+A1)*(4+A1)		252
2+1*A1)*(1+A1)^2*(2+A1)*(4+A1)		16
2+3+2*A1)*(1+A1)^2*(3+A1)*(3+A1)+(1+3+2*A1)*(2+A1)^2*(3+A1)*(3+A1)		189
2+1*A1)*(1+A1)^2*(2+A1)*(3+A1)		12
2+1*A1)*(1+A1)^2*(2+A1)*(2+A1)		8
1+1*A1)*(2+A1)*(5+A1)*(5+A1)*(5+A1)+(1+2+2*A1)*(3+A1)*(5+A1)*(5+A1)*(5+A1)+(1+2+3+3*A1)*(4+A1)*(5+A1)		4375
1+1*A1)*(2+A1)*(4+A1)*(4+A1)*(5+A1)+(1+2+2*A1)*(3+A1)*(4+A1)*(6+A1)		880
1+1*A1)*(2+A1)*(3+A1)*(3+A1)*(5+A1)		90
1+1*A1)*(2+A1)*(4+A1)*(5+A1)*(5+A1)+(1+2+2*A1)*(3+A1)*(4+A1)*(5+A1)*(5+A1)		1100
1+1*A1)*(2+A1)*(3+A1)*(4+A1)*(5+A1)		120
1+1*A1)*(2+A1)*(3+A1)*(5+A1)*(5+A1)		150
1+1*A1)*(2+A1)*(4+A1)*(4+A1)*(4+A1)+(1+2+2*A1)*(3+A1)*(4+A1)*(4+A1)		704
1+1*A1)*(2+A1)*(3+A1)*(3+A1)*(4+A1)		72
1+1*A1)*(2+A1)*(3+A1)*(4+A1)*(4+A1)		96
1+1*A1)*(2+A1)*(3+A1)*(3+A1)*(3+A1)		54
42525		4252