Finding Sum of the Series and power sum of numbers, "*Nth difference sequences*"

Basis on "*signed Stirling numbers of the first kind*".

In this article we are going to introduce a General formula for finding the Sums of Series and power sum of all type of numbers in Arithmetic Sequences in form of {Nth difference sequences}.

Since the sequences as { $1^k, 2^k, 3^k, \ldots$ }, or in general case as, { $a^k, (a+b)^k, (a+2b)^k, (a+3b)^k, \ldots, (a+nb)^k$ }, are types of (Nth difference sequences), therefore the introduced formula can be use for calculating the power sum of all type of numbers as, $1^k + 2^k + 3^k + \ldots + n^k$ or in general case $a^k + (a+b)^k + (a+2b)^k + (a+3b)^k + \ldots + (a+nb)^k$

Floor No.f	f = ?	$(a_f)_1$	$(a_f)_2$	$(a_f)_3$	$(a_f)_4$	$(a_f)_5$	$(a_f)_6$	$(a_f)_7$	$(a_f)_8$	$(a_f)_9$	$(a_f)_{10}$	…	$(a_f)_t$
…													
Floor No.8	f = 8	$(a_8)_1$	$(a_8)_2$	$(a_8)_3$	$(a_8)_4$	$(a_8)_5$	$(a_8)_6$	$(a_8)_7$	$(a_8)_8$	$(a_8)_9$	$(a_8)_{10}$		
Floor No.7	f = 7	$(a_7)_1$	$(a_7)_2$	$(a_7)_3$	$(a_7)_4$	$(a_7)_5$	$(a_7)_6$	$(a_7)_7$	$(a_7)_8$	$(a_7)_9$			
Floor No.6	f = 6	$(a_6)_1$	$(a_6)_2$	$(a_6)_3$	$(a_6)_4$	$(a_6)_5$	$(a_6)_6$	$(a_6)_7$	$(a_6)_8$				
Floor No.5	f = 5	$(a_5)_1$	$(a_5)_2$	$(a_5)_3$	$(a_5)_4$	$(a_5)_5$	$(a_5)_6$	$(a_5)_7$					
Floor No.4	f = 4	$(a_4)_1$	$(a_4)_2$	$(a_4)_3$	$(a_4)_4$	$(a_4)_5$	$(a_4)_6$						
Floor No.3	f = 3	$(a_3)_1$	$(a_3)_2$	$(a_3)_3$	$(a_3)_4$	$(a_3)_5$							
Floor No.2	f = 2	$(a_2)_1$	$(a_2)_2$	$(a_2)_3$	$(a_2)_4$								
FloorNo.1	f = 1 Common difference floor	$(a_1)_1$	$(a_1)_2$	$(a_1)_3$									
		0	0										

And a numerical sample for above {**fth differences sequences**}`s building, is as down figure. ↓

Floor No.f	f = ?	$(a_f)_1$	$(a_f)_2$	$(a_f)_3$	$(a_f)_4$	$(a_f)_5$	$(a_f)_6$	$(a_f)_7$	$(a_f)_8$	$(a_f)_9$	$(a_f)_{10}$	…	$(a_f)_t$
…												…	
Floor No.8	f = 8	–45	–46	–70	–108	–61	455	2484	8345	22547	53064		
Floor No.7	f = 7	–1	–24	–38	47	516	2029	5861	14202	30517			
Floor No.6	f = 6	–23	–14	85	469	1513	3832	8341	16315				
Floor No.5	f = 5	9	99	384	1044	2319	4509	7974					
Floor No.4	f = 4	90	285	660	1275	2190	3465						
Floor No.3	f = 3	195	375	615	915	1275							
Floor No.2	f = 2	180	240	300	360								
Floor No.1	f = 1 Common difference floor	60	60	60									
		0	0										

In above figures, each one of the sequences makes a floor of building; and the difference of the terms on created floor makes a sequence in lower floor of it.

In this article we are going to introduce a formula for summation of the series in above kinds of arithmetic sequences, basis on coefficients of “***signed Stirling numbers of the first kind***”.

In mentioned formulas, $\left(a_f\right)_t$ show the place of terms, in building of the sequences.

Element “a” means arithmetic sequence, and the value of “f” is number of the floor which the arithmetic sequences are located on it. and the value of “t” is the number of the term or first “t” terms for summation.

Attention: for the reason that, there is similarity in signs of elements in ***sequence*** and ***Stirling numbers***. for preventing of interference between subjects, we put some changes in signs of elements in **sequence`s** subject. as “a” for sequence, and “f” means number of floor, and “t” means time of term in sequence.

For example: $\left(a_5\right)_7$ is the seventh 7th term of the arithmetic sequence No.5 which located in floor No.5 or (f= 5 & t=7) and $\sum_{t=1}^{7}\left(a_5\right)_7$ means the summation of the first 7 terms of arithmetic

Series. $\sum_{t=1}^{7}\left(a_5\right)_7 = 9 + 99 + 384 + 1044 + 2319 + 4509 + 7974 = 1633$

The formula for summation of the first “t” terms of the series in arithmetic sequences. ↓

$$\sum_{t=1}^{t}\left(a_f\right)_t = \left[\left(a_1\right)_1 \cdot \frac{\sum_{k=1,\,f=1}^{n,\,k=(f-0)}\left[s(n,k)\cdot t^{(f-0)}\right]}{(f-0)!}\right] + \left[\left(a_2\right)_1 \cdot \frac{\sum_{k=1,\,f=2}^{n,\,k=(f-1)}\left[s(n,k)\cdot t^{(f-1)}\right]}{(f-1)!}\right] + \left[\left(a_3\right)_1 \cdot \frac{\sum_{k=1,\,f=3}^{n,\,k=(f-2)}\left[s(n,k)\cdot t^{(f-2)}\right]}{(f-2)!}\right] + \dots + \left[\left(a_f\right)_1 \cdot \frac{\sum_{k=1,\,f=f}^{n,\,k=[f-(f-1)]}\left[s(n,k)\cdot t^{[f-(f-1)]}\right]}{[f-(f-1)]!}\right]$$

In above main formula, the first “*fractional expression*” is an equation in term of “t”, degree of “(f-0)”, and the values of the parameters are, a set of “***signed Stirling numbers of the first kind***” located in row No, “f-0”, on Stirling triangular array of the first kind.

For example: the first “*fractional expression*” in above formula for sequence $\left(a_7\right)_t$ is. ↓

$$\sum_{t=1}^{t}\left(a_f\right)_t = \left[\left(a_1\right)_1 \cdot \frac{\sum_{k=1,\,f=1}^{n,\,k=(f-0)}\left[s(n,k)\cdot t^{(f-0)}\right]}{(f-0)!}\right] + \dots \;\rightarrow\; \sum_{t=1}^{t}\left(a_7\right)_t = \left[\left(a_1\right)_1 \cdot \frac{\sum_{k=1,\,f=1}^{n,\,k=(7-0)}\left[s\left[n_7, k_{(1,2,3,\dots,7)}\right]\cdot t^{(7-0)}\right]}{(7-0)!}\right] + \dots$$

Signed Stirling numbers of the first kind:

$n_{\blacksquare}$ \ $k_{\blacksquare}$	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
n_1	1								
n_2	−1	1							
n_3	2	−3	1						
n_4	−6	11	−6	1					
n_5	24	−50	35	−10	1				
n_6	−120	274	−225	85	−15	1			
n_7	720	−1764	1624	−735	175	−21	1		
n_8	−5040	13068	−13132	6769	−1960	322	−28	1	
n_9	40320	−109584	118124	−67284	22449	−4536	546	−36	1

The first "*fractional expression*" in above "*main formula*" for sequence $(a_7)_t$ is. ↓

$$\sum_{t=\blacksquare}^{t}(a_f)_t=(a_1)_1\cdot\left[\frac{s(n_7,k_1)\cdot t^{(f_1-0)}+s(n_7,k_2)t^{(f_2-0)}+s(n_7,k_3)t^{(f_3-0)}+s(n_7,k_4)t^{(f_4-0)}+s(n_7,k_5)t^{(f_5-0)}+s(n_7,k_6)t^{(f_6-0)}+s(n_7,k_7)t^{(f_7-0)}}{(f-0)!}\right]+\blacksquare$$

Then after putting the related "*Stirling numbers*" as parameters of equation, the first "*fractional expression*" will be create for sequence $(a_7)_t$ as ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t=(a_1)_1\cdot\left(\frac{720t^1-1764t^2+1624t^3-735t^4+175t^5-21\cdot t^6+1\cdot t^7}{7!}\right)+\blacksquare$$

The second "*fractional expression*" in above *formula,* for sequence $(a_7)_t$ is. ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t=(a_1)_1\cdot(\text{First fraction})+(a_2)_1\cdot\left[\frac{s(n_6,k_1)\cdot t^{(f_1-1)}+s(n_6,k_2)t^{(f_2-1)}+s(n_6,k_3)t^{(f_3-1)}+s(n_6,k_4)t^{(f_4-1)}+s(n_6,k_5)t^{(f_5-1)}+s(n_6,k_6)t^{(f_6-1)}}{(f-1)!}\right]$$

Then after putting the related "*Stirling numbers*" as parameters of equation, the second "*fractional expression*" will be create for sequence $(a_7)_t$ as ↓↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot \left[\frac{-120t^{(2-1)} + 274t^{(3-1)} - 225t^{(4-1)} + 85t^{(5-1)} - 15t^{(6-1)} + 1 \cdot t^{(7-1)}}{(7-1)!}\right]$$

$$\left[\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot \left(\frac{-120t + 274t^2 - 225t^3 + 85t^4 - 15t^5 + 1 \cdot t^6}{6!}\right)\right] + \blacksquare$$

The third "*fractional expression*" in above *formula,* for sequence $(a_7)_t$ is ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot (\text{Second fraction}) + (a_3)_1 \cdot \left[\frac{s(n_5,k_1) \cdot t^{(f_1-2)} + s(n_5,k_2)t^{(f_2-2)} + s(n_5,k_3)t^{(f_3-2)} + s(n_5,k_4)t^{(f_4-2)} + s(n_5,k_5)t^{(f_5-2)}}{(f-2)!}\right] + \blacksquare$$

Then after putting the related "*Stirling numbers*" as parameters of equation the third "*fractional expression*" will be create for sequence $(a_7)_t$ as ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot (\text{Second fraction}) + (a_3)_1 \cdot \left[\frac{24t^{(f_1-2)} - 50t^{(f_2-2)} + 35t^{(f_3-2)} - 10t^{(f_4-2)} + 1 \cdot t^{(f_5-2)}}{(f-2)!}\right] + \ldots + \blacksquare$$

And then ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot (\text{Second fraction}) + (a_3)_1 \cdot \left(\frac{24t - 50t^2 + 35t^3 - 10t^4 + t^5}{5!}\right) + \ldots + \blacksquare$$

Now we are completed the third "*fractional expression*" and in continue by above mentioned manner, (by reducing or increasing the lower and upper limits of sigma`s parameters, as value of "f, n, k" we should create the other "*fractional expression*" one by one till the last one of them.

The last "*fractional expression*" in above *formula,* for sequence $(a_7)_t$ is ↓

$$\sum_{t=\blacksquare}^{t}(a_7)_t = (a_1)_1 \cdot (\text{First fraction}) + (a_2)_1 \cdot (\text{Second fraction}) + (a_3)_1 \cdot (\text{Third fraction}) + \ldots + (a_f)_1 \cdot \frac{\sum_{k=1, f=f}^{n,k=[f-(f-1)]}\left[s(n,k) \cdot t^{[f-(f-1)]}\right]}{[f-(f-1)]!}$$

Then after putting the related "*Stirling numbers*" as parameters of equation the last "fractional expression,(*for present example, "7th* expression") will be create for sequence $(a_7)_t$ as ↓↓

$$\sum_{t=1}^{t}(a_7)_t=(a_1)_1\cdot(\text{First fraction})+(a_2)_1\cdot(\text{Second fraction})+(a_3)_1\cdot(\text{Third fraction})+\ldots+(a_7)_1\cdot\frac{\displaystyle\sum_{k=1,\,f=7}^{n,k=[7-(7-1)]}\left[s(1,1)\cdot t^{[7-(7-1)]}\right]}{[7-(7-1)]!}$$

$$\sum_{t=1}^{t}(a_7)_t=(a_1)_1\cdot(\text{1th fraction})+(a_2)_1\cdot(\text{2th fraction})+(a_3)_1\cdot(\text{3th fraction})+(a_4)_1\cdot(\text{4th fraction})+(a_5)_1\cdot(\text{5th fraction})+(a_6)_1\cdot(\text{6th fraction})+(a_7)_1\cdot\frac{1\cdot t^1}{1!}$$

The general fourmula for summation of the first “t” terms of the series of arithmetic sequences $(a_7)_t$ locates in floor No.7 with a sample for sequence $(a_7)_8$ is ↓

$$\sum_{t=1}^{t}(a_7)_t=(a_1)_1\cdot\left(\frac{720t-1764t^2+1624t^3-735t^4+175t^5-21\cdot t^6+t^7}{7!}\right)+(a_2)_1\cdot\left(\frac{-120t+274t^2-225t^3+85t^4-15t^5+t^6}{6!}\right)+(a_3)_1\cdot\left(\frac{24t-50t^2+35t^3-10t^4+t^5}{5!}\right)+(a_4)_1\cdot\left(\frac{-6\cdot t+11\cdot t^2-6\cdot t^3+t^4}{4!}\right)+(a_5)_1\cdot\left(\frac{2\cdot t-3\cdot t^2+t^3}{3!}\right)+(a_6)_1\cdot\left(\frac{-t+t^2}{2!}\right)+(a_7)_1\cdot\left(\frac{t}{1!}\right)=$$

$$-1+-24+-38+47+516+2029+5861+14202=22592$$

$$60\left(\frac{720\cdot 8-1764\cdot 8^2+1624\cdot 8^3-735\cdot 8^4+175\cdot 8^5-21\cdot 8^6+8^7}{7!}\right)+180\left(\frac{-120\cdot 8+274\cdot 8^2-225\cdot 8^3+85\cdot 8^4-15\cdot 8^5+8^6}{6!}\right)+195\left(\frac{24\cdot 8-50\cdot 8^2+35\cdot 8^3-10\cdot 8^4+8^5}{5!}\right)+90\left(\frac{-6\cdot 8+11\cdot 8^2-6\cdot 8^3+8^4}{4!}\right)+9\cdot\left(\frac{2\cdot 8-3\cdot 8^2+8^3}{3!}\right)+(-23)\cdot\left(\frac{-8+8^2}{2!}\right)+(-1)\cdot\left(\frac{8}{1!}\right)=22592$$

By the main formula we can create the formulas for other sequences located in lower and upper floors

For example: the formula for the sequence $(a_8)_t$ and an example for $(a_8)_{10}$ are as. ↓↓

$$\sum_{t=1}^{t}(a_8)_t=(a_1)_1\cdot\left(\frac{-5040t+13068t^2-13132t^3+6769t^4-1960t^5+322t^6-28\cdot t^7+t^8}{8!}\right)+(a_2)_1\cdot\left(\frac{720t-1764t^2+1624t^3-735t^4+175t^5-21\cdot t^6+t^7}{7!}\right)+(a_3)_1\cdot\left(\frac{-120t+274t^2-225t^3+85\cdot t^4-15\cdot t^5+t^6}{6!}\right)+(a_4)_1\cdot\left(\frac{24\cdot t-50\cdot t^2+35\cdot t^3-10\cdot t^4+t^5}{5!}\right)+(a_5)_1\cdot\left(\frac{-6\cdot t+11\cdot t^2-6\cdot t^3+t^4}{4!}\right)+(a_6)_1\cdot\left(\frac{2\cdot t-3\cdot t^2+t^3}{3!}\right)+(a_7)_1\cdot\left(\frac{-t+t^2}{2!}\right)+(a_8)_1\cdot\left(\frac{t}{1!}\right)=$$

$$(-45)+(-46)+(-70)+(-108)+(-61)+(455)+(2484)+(8345)+(22547)+(53064)=86565$$

$$60\left(\frac{10^8-28\times 10^7+322\times 10^6-1960\times 10^5+6769\times 10^4-13132\times 10^3+13068\times 10^2-5040\times 10}{8!}\right)+180\left(\frac{10^7-21\times 10^6+175\times 10^5-735\times 10^4+1624\times 10^3-1764\times 10^2+720\times 10}{7!}\right)+195\left(\frac{10^6-15\times 10^5+85\times 10^4-225\times 10^3+274\,10^2-120\times 10}{6!}\right)+90\left(\frac{10^5-10\times 10^4+35\times 10^3-50\times 10^2+24\cdot 10}{5!}\right)+9\cdot\left(\frac{10^4-6\times 10^3+11\times 10^2-6\times 10}{4!}\right)+-23\cdot\left(\frac{10^3-3\times 10^2+2\times 10}{3!}\right)+-1\cdot\left(\frac{10^2-10}{2!}\right)+-45\cdot\left(\frac{10}{1!}\right)=86565$$

For example: the formula for the sequence $(a_9)_t$ and an example for $(a_9)_8$ are as below. ↓↓

$$\sum_{t=1}^{8}(a_9)_8=\left[(a_1)_1\cdot\frac{\left[\displaystyle\sum_{f=1,k=1}^{n,k,(f-0)=(9-0)}\left[s(9,k)\cdot 8^{(9-0)}\right]\right]}{(9-0)!}\right]+\left[\left[a_{(2)}\right]_1\cdot\frac{\left[\displaystyle\sum_{f=2,k=1}^{n,k,(f-1)=(9-1)}\left[s(8,k)\cdot 8^{(9-1)}\right]\right]}{(9-1)!}\right]+\left[\left[a_{(3)}\right]_1\cdot\frac{\left[\displaystyle\sum_{f=3,k=1}^{n,k,(f-2)=(9-2)}\left[s(7,k)\cdot 8^{(9-2)}\right]\right]}{(9-2)!}\right]+\ldots+\left[(a_9)_1\cdot\frac{\left[\displaystyle\sum_{f=9,k=1}^{n,k,[f-(f-1)]=[9-(9-1)]}\left[s(1,k)\cdot 8^{[9-(9-1)]}\right]\right]}{[9-(9-1)]!}\right]$$

$$60\left(\frac{40320\cdot 8-109584\cdot 8^2+118124\cdot 8^3-67284\cdot 8^4+22449\cdot 8^5-4536\cdot 8^6+546\cdot 8^7-36\cdot 8^8+8^9}{9!}\right)+180\left(\frac{-5040\cdot 8+13068\cdot 8^2-13132\cdot 8^3+6769\cdot 8^4-1960\cdot 8^5+322\cdot 8^6-28\cdot 8^7+8^8}{8!}\right)+195\left(\frac{720\cdot 8-1764\cdot 8^2+1624\cdot 8^3-735\cdot 8^4+175\cdot 8^5-21\cdot 8^6+8^7}{7!}\right)+90\left(\frac{-120\cdot 8+274\cdot 8^2-225\cdot 8^3+85\cdot 8^4-15\cdot 8^5+8^6}{6!}\right)+9\left(\frac{24\cdot 8-50\cdot 8^2+35\cdot 8^3-10\cdot 8^4+8^5}{5!}\right)+(-23)\cdot\left(\frac{-6\cdot 8+11\cdot 8^2-6\cdot 8^3+8^4}{4!}\right)+(-1)\cdot\left(\frac{2\cdot 8-3\cdot 8^2+8^3}{3!}\right)+(-45)\cdot\left(\frac{-8+8^2}{2!}\right)+(-0.2)\cdot\left(\frac{8}{1!}\right)=1836.4$$

Important point: by adding the value of the first term of upper Sequence to formula of lower sequence we can determine the terms of upper sequences.

Calculating the power sum of natural numbers as, $1^k+2^k+3^k+\ldots+n^k$ **or in general case as,**

$a^k,(a+b)^k,(a+2b)^k,(a+3b)^k,\ldots,(a+nb)^k$

Below is a sample of the building for sequences { $1^7,2^7,3^7,\ldots,10^7$ } ↓

Floor No.8 f = 8	1^7	2^7	3^7	4^7	5^7	6^7	7^7	8^7	9^7	10^7
Floor No.7 f = 7	127	2059	14197	61741	201811	543607	1273609	2685817	5217031	
Floor No.6 f = 6	1932	12138	47544	140070	341796	730002	1412208	2531214		
Floor No.5 f = 5	10206	35406	92526	201726	388206	682206	1119006			
Floor No.4 f = 4	25200	57120	109200	186480	294000	436800				
Floor No.3 f = 3	31920	52080	77280	107520	142800					
Floor No.2 f = 2	20160	25200	30240	35280						
Floor No.1 f = 1 Common difference floor	5040 = 7!	5040 = 7!	5040 = 7!							

For calculating the **power sum of the natural** numbers as { $1^7 + 2^7 + 3^7 + \ldots + 10^7$ } we should use of the formula which it has created for sequences located in { Floor No. 8 }

$$\sum_{n=1}^{n} (a_8)_n = (a_1)_1 \cdot \left(\frac{n^8 - 28n^7 + 322n^6 - 1960n^5 + 6769n^4 - 13132n^3 + 13068n^2 - 5040n}{8!}\right) + (a_2)_1 \cdot \left(\frac{n^7 - 21n^6 + 175n^5 - 735n^4 + 1624n^3 - 1764n^2 + 720n}{7!}\right) + (a_3)_1 \cdot \left(\frac{n^6 - 15n^5 + 85n^4 - 225n^3 + 274n^2 - 120n}{6!}\right) + (a_4)_1 \cdot \left(\frac{n^5 - 10n^4 + 35n^3 - 50n^2 + 24n}{5!}\right) + (a_5)_1 \cdot \left(\frac{n^4 - 6n^3 + 11n^2 - 6n}{4!}\right) + (a_6)_1 \cdot \left(\frac{n^3 - 3n^2 + 2n}{3!}\right) + (a_7)_1 \cdot \left(\frac{n^2 - n}{2!}\right) + (a_8)_1 \cdot \left(\frac{n}{1!}\right) =$$

$$5040\left(\frac{10^8 - 28\times 10^7 + 322\times 10^6 - 1960\times 10^5 + 6769\times 10^4 - 13132\times 10^3 + 13068\times 10^2 - 5040\times 10}{8!}\right) + 20160\left(\frac{10^7 - 21\times 10^6 + 175\times 10^5 - 735\times 10^4 + 1624\times 10^3 - 1764\times 10^2 + 720\times 10}{7!}\right) + 31920\left(\frac{10^6 - 15\times 10^5 + 85\times 10^4 - 225\times 10^3 + 274 10^2 - 120\times 10}{6!}\right) + 25200\left(\frac{10^5 - 10\times 10^4 + 35\times 10^3 - 50\times 10^2 + 24 10}{5!}\right) + 10206\left(\frac{10^4 - 6\times 10^3 + 11\times 10^2 - 6\times 10}{4!}\right) + 1932\left(\frac{10^3 - 3\times 10^2 + 2\times 10}{3!}\right) + 127\left(\frac{10^2 - 10}{2!}\right) + 1\cdot\left(\frac{10}{1!}\right) = 18080425$$

$$1^7 + 2^7 + 3^7 + 4^7 + 5^7 + 6^7 + 7^7 + 8^7 + 9^7 + 10^7 = 18080425$$

For calculating the **power sum of the numbers** as sequences { $a^k, (a + b)^k, (a + 2b)^k, (a + 3b)^k, \ldots, (a + nb)^k$ } or In The form of the series as, $a^k + (a + b)^k + (a + 2b)^k + (a + 3b)^k + \ldots + (a + nb)^k$

we should use of the formula,of the sequences locates in { Floor No. 8 } of sequences\`s building as ↓

Floor No.8 f = 8	3^7	5^7	7^7	9^7	11^7	13^7	15^7	17^7	19^7	21^7
Floor No.7 f = 7	75938	745418	3959426	14704202	43261346	108110858	239479298	483533066	907216802	
Floor No.6 f = 6	669480	3214008	10744776	28557144	64849512	131368440	244053768	423683736		
Floor No.5 f = 5	2544528	7530768	17812368	36292368	66518928	112685328	179629968			
Floor No.4 f = 4	4986240	10281600	18480000	30226560	46166400	66944640				
Floor No.3 f = 3	5295360	8198400	11746560	15939840	20778240					
Floor No.2 f = 2	2903040	3548160	4193280	4838400						
Floor No.1 f = 1 Common difference floor	645120	645120	645120							

For example: the power sum of sequece { $3^7, 5^7, 7^7, \ldots, 11^7$ } in form of { $3^7, (3 + 2)^7, (3 + 4)^7, (3 + 6)^7, \ldots, (3 + 8)^k$ } or Series $3^7 + 5^7 + 7^7 + \ldots + 11^7$ in form of $a + (a + b)^k + (a + 2b)^k + (a + 3b)^k + \ldots + (a + nb)^k$ is as below ↓

$$\sum_{n=1}^{n}(a_8)_n = (a_1)_1\cdot\left(\frac{n^8-28n^7+322n^6-1960n^5+6769n^4-13132n^3+13068n^2-5040n}{8!}\right)+(a_2)_1\cdot\left(\frac{n^7-21n^6+175n^5-735n^4+1624n^3-1764n^2+720n}{7!}\right)+(a_3)_1\cdot\left(\frac{n^6-15n^5+85n^4-225n^3+274n^2-120n}{6!}\right)+(a_4)_1\cdot\left(\frac{n^5-10n^4+35n^3-50n^2+24n}{5!}\right)+(a_5)_1\cdot\left(\frac{n^4-6n^3+11n^2-6n}{4!}\right)+(a_6)_1\cdot\left(\frac{n^3-3n^2+2n}{3!}\right)+(a_7)_1\cdot\left(\frac{n^2-n}{2!}\right)+(a_8)_1\cdot\left(\frac{n}{1!}\right)=$$

$$645120\left(\frac{5^8-28\times5^7+322\times5^6-1960\times5^5+6769\times5^4-13132\times5^3+13068\times5^2-5040\times5}{8!}\right)+2903040\left(\frac{5^7-21\times5^6+175\times5^5-735\times5^4+1624\times5^3-1764\times5^2+720\times5}{7!}\right)+5295360\left(\frac{5^6-15\times5^5+85\times5^4-225\times5^3+2745^2-120\times5}{6!}\right)+4986240\left(\frac{5^5-10\times5^4+35\times5^3-50\times5^2+245}{5!}\right)+2544528\left(\frac{5^4-6\times5^3+11\times5^2-6\times5}{4!}\right)+669480\left(\frac{5^3-3\times5^2+2\times5}{3!}\right)+75938\left(\frac{5^2-5}{2!}\right)+3^7\cdot\left(\frac{5}{1!}\right)=25173995$$

$$3^7+5^7+7^7+9^7+11^7=25173995$$

By above mentioned formula we can calculate power sum of natural or decimal or other types of numbers; only on a condition that, the terms of power sum series can make a sequences in form of "Nth difference sequence" as below"*sequence*" ↓

For example power sum of series as $3.3^6+5.4^6+7.5^6+9.6^6+11.7^6$

$$(1.2+2.1)^6+(3.3+2.1)^6+(5.4+2.1)^6+(7.5+2.1)^6+(9.6+2.1)^6=3551986.88635499$$

$$61751.60711996781\left(\frac{5^7-21\times5^6+175\times5^5-735\times5^4+1624\times5^3-1764\times5^2+720\times5}{7!}\right)+251417.25756000658\left(\frac{5^6-15\times5^5+85\times5^4-225\times5^3+2745^2-120\times5}{6!}\right)+404115.95951999945\left(\frac{5^5-10\times5^4+35\times5^3-50\times5^2+245}{5!}\right)+321915.50873999996\left(\frac{5^4-6\times5^3+11\times5^2-6\times5}{4!}\right)+129680.16100199998\left(\frac{5^3-3\times5^2+2\times5}{3!}\right)+23503.44332700008\left(\frac{5^2-5}{2!}\right)+1291.467968999996\left(\frac{5}{1!}\right)=3551986.886354998$$

3.3^6 5.4^6 7.5^6 9.6^6 11.7^6 13.8^6 15.9^6

23503.44332700008 153183.604329 604779.2740709999 1782406.412072994 4341598.235415004 9251056.825856995

129680.16100199998 451595.66974199994 1177627.1380019994 2559191.823342005 4909458.590441991

321915.50873999996 726031.4682599994 1381564.6853400054 2350266.767099986

404115.95951999945 655533.2170800006 968702.0817599804

251417.25756000658 313168.86467997440

61751.60711996781

The above exampled series is consisted of five terms, and for calculating the sum of other times of terms we must change the values of "n" in formula.

For example:

Sum of "7" term of above mentioned series is. ↓

$$61751.60711996781\left(\frac{7^7-21\times7^6+175\times7^5-735\times7^4+1624\times7^3-1764\times7^2+720\times7}{7!}\right)+251417.25756000658\left(\frac{7^6-15\times7^5+85\times7^4-225\times7^3+2747^2-120\times7}{6!}\right)+404115.95951999945\left(\frac{7^5-10\times7^4+35\times7^3-50\times7^2+247}{5!}\right)+321915.50873999996\left(\frac{7^4-6\times7^3+11\times7^2-6\times7}{4!}\right)+129680.16100199998\left(\frac{7^3-3\times7^2+2\times7}{3!}\right)+23503.44332700008\left(\frac{7^2-7}{2!}\right)+1291.467968999996\left(\frac{7}{1!}\right)=2661656 8.58657999$$

$$3.3^6+5.4^6+7.5^6+9.6^6+11.7^6+13.8^6+15.9^6=26616568.58655$$

Key words:

Series, Nth difference sequences, power sum of numbers, sequences, Stirling numbers

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1- Wikipedia “Stirling numbers of the first kind”
2- OEIS.