Stirling numbers & powers (exponent) & factorials

Explicit formulas

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There are two explicit formulas to show how the relations of the Stirling numbers of the second kind and powers (exponents) and factorial numbers, as down figures

Formula 1:

$$S(n+1,1)\cdot\left[\frac{(x-1)!}{(x-1)!}\right]+S(n+1,2)\cdot\left[\frac{(x-1)!}{(x-2)!}\right]+\ldots+S(n+1,n+1)\cdot\left[\frac{(x-1)!}{[x-(n+1)]!}\right]=x^{n} \qquad x>n$$

For example: $x^{n}=12^{5}$ & $x>n$ & related Stirling numbers as coefficients of factorial fractions

$$S(n+1,k)=S(5+1,k)=(0,1,31,90,65,15,1)$$

$$1\cdot\left[\frac{(12-1)!}{(12-1)!}\right]+31\cdot\left[\frac{(12-1)!}{(12-2)!}\right]+90\cdot\left[\frac{(12-1)!}{(12-3)!}\right]+65\cdot\left[\frac{(12-1)!}{(12-4)!}\right]+15\cdot\left[\frac{(12-1)!}{(12-5)!}\right]+1\cdot\left[\frac{(12-1)!}{(12-6)!}\right]=248832 \qquad 12^{5}=248832$$

Formula 2:

$$S(n+1,1)\cdot\left[\frac{(x-1)!}{(x-1)!}\right]+S(n+1,2)\cdot\left[\frac{(x-1)!}{(x-2)!}\right]+\ldots+S(n+1,x)\cdot\left[\frac{(x-1)!}{(x-x)!}\right]=x^{n} \qquad x\le n$$

For example: $x^{n}=5^{9}$ & $x\le n$ & related Stirling numbers as coefficients of factorial fractions

$$S(n+1,k)=S(9+1,k)=(0,1,511,9330,34105,42525,22827,5880,750,45,1)$$

$$1\cdot\left[\frac{(5-1)!}{(5-1)!}\right]+511\cdot\left[\frac{(5-1)!}{(5-2)!}\right]+9330\cdot\left[\frac{(5-1)!}{(5-3)!}\right]+34105\cdot\left[\frac{(5-1)!}{(5-4)!}\right]+42525\cdot\left[\frac{(5-1)!}{(5-5)!}\right]=1953125 \qquad 5^{9}=1953125$$

Key words: Stirling numbers; powers or exponents; factorials; formula for relation of the Stirling numbers, powers, factorials;